

POS: R-801

GR20: X-1920 Y-1850

REF: LC-2803-TP-6L

Concrete Penetration Dynamics: The A4 Pipeline

Translating Li & Chen (2003) from Theoretical Physics to Executable Logic.

The Structural Limits of Legacy Empiricism

Legacy vs. Modern

Unit Dependency	Legacy models are mathematically brittle. The NDRC formula relies on a magic coefficient (3.8×10^{-5}) that breaks completely if inputs are switched from SI to Imperial units.
Ambiguous Geometry	Legacy models force nose shapes into discrete, rigid buckets (NDRC flat = 0.72 , blunt = 0.84 , spherical = 1.0 , sharp = 1.14). They cannot calculate intermediate, truncated, or eroded shapes.
Narrow Validity	Calibrated for shallow impacts ($0.6 < X/d < 2.0$). Fails completely at deep penetrations ($X/d > 5$) and velocities exceeding 500 m/s .

⚠ NDRC: $G = 3.8 \times 10^{-5} \frac{N \cdot M}{d^3} (V_i)^{1.8}$

⚠ Barr: $\frac{x}{d} = 1.23 \cdot \ln \left(1 + \frac{V}{1200} \right)^2$

⚠ ACE: $P = \frac{K \cdot W \cdot V}{A}$

The Dual-Theory Paradigm of Li & Chen (2003)

Dimensional Analysis

Uses Buckingham Pi theorem to distill 10 chaotic physical variables into three pure dimensionless numbers. Solves the unit-dependency problem.



10 Variables \rightarrow 3 Dimensionless Numbers
Buckingham Pi: $f(\pi_1, \pi_2, \pi_3) = 0$

Dynamic Cavity Expansion

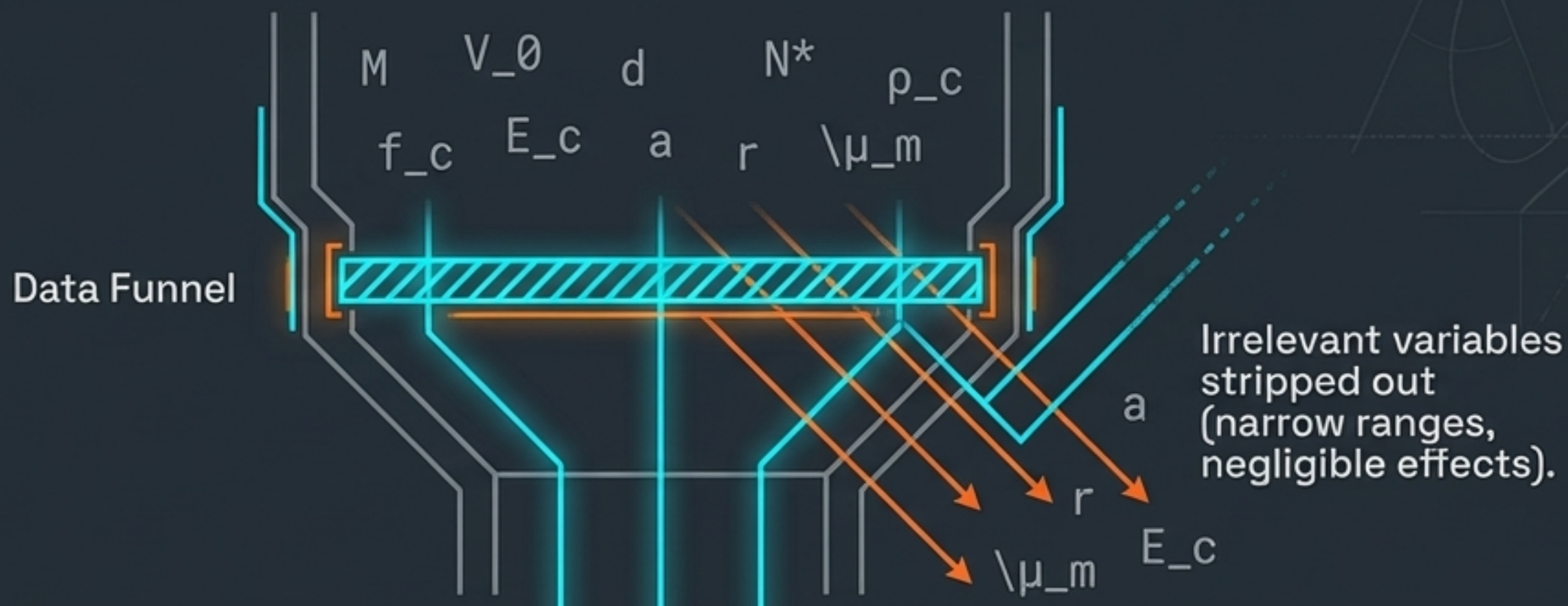
Analyzes the precise physical forces at the projectile tip. Replaces discrete geometry buckets with a continuous mathematical integral.



Integral Force Analysis
 $\int (\text{Continuous Geometry}) dX > \sum (\text{Discrete Buckets})$

Synthesis Node: A fully deterministic, dimensionless penetration framework valid from $X/d = 0.07$ to $X/d = 92.8$.

Filtering the Noise via Buckingham Pi



$$1 \quad I^* = \frac{M \cdot V_0^2}{d^3 \cdot f_c}$$

Impact Factor: Ratio of projectile kinetic energy to the target's volumetric capacity.

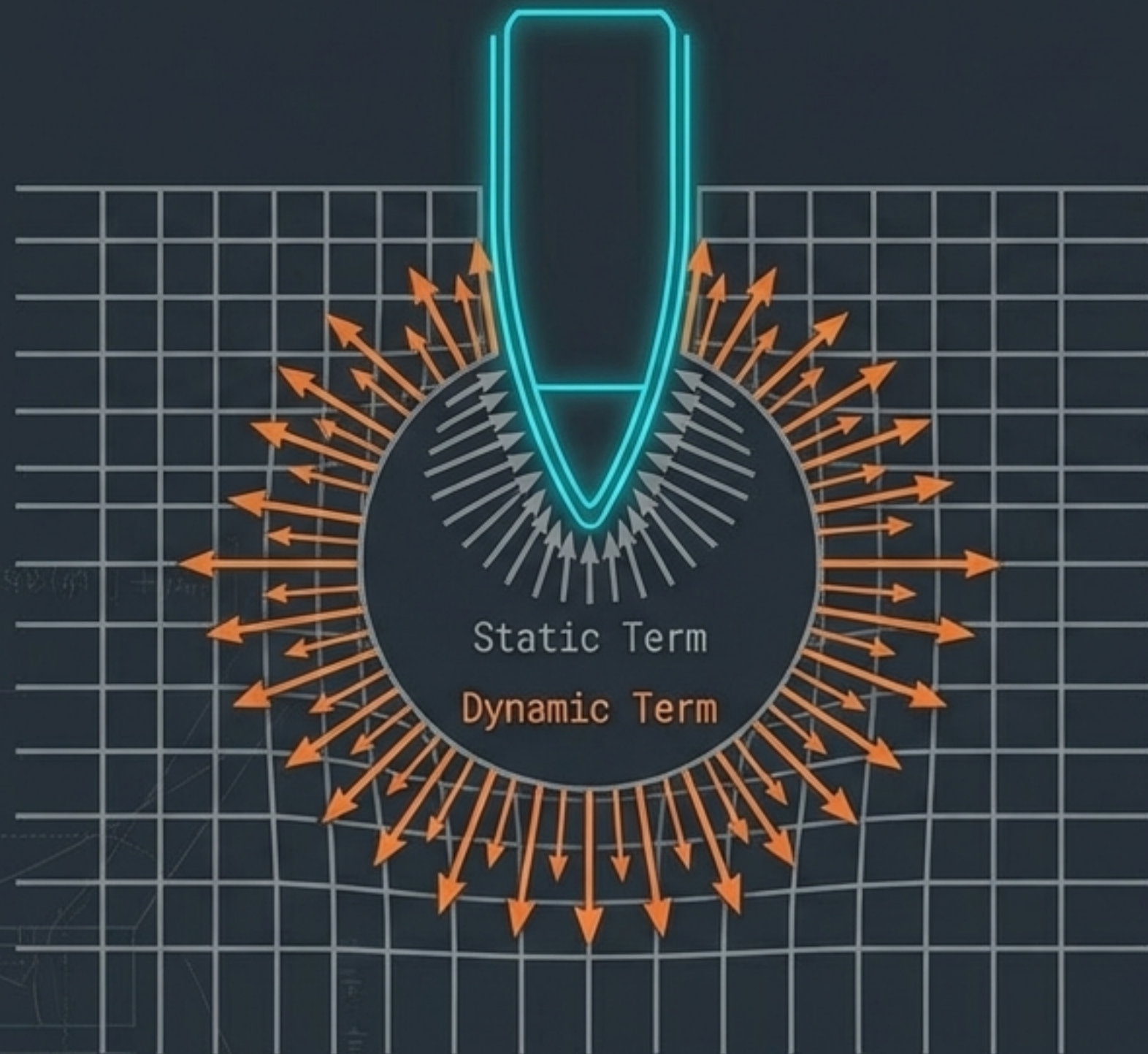
$$2 \quad \lambda = \frac{M}{\rho_c \cdot d^3}$$

Mass Ratio: Combines projectile slenderness and relative density.

$$3 \quad N^*$$

Nose Factor: The geometric efficiency of the projectile tip.

The Physics of Dynamic Cavity Expansion



Force Equation

$$F = \left(\frac{\pi d^2}{4} \right) * (S f_c + N^* \rho_c V^2)$$

S_c

Static Term ($S f_c$)

The concrete's shear strength under confinement. Independent of velocity and independent of the nose shape.



Dynamic Term ($N^* \rho_c V^2$)

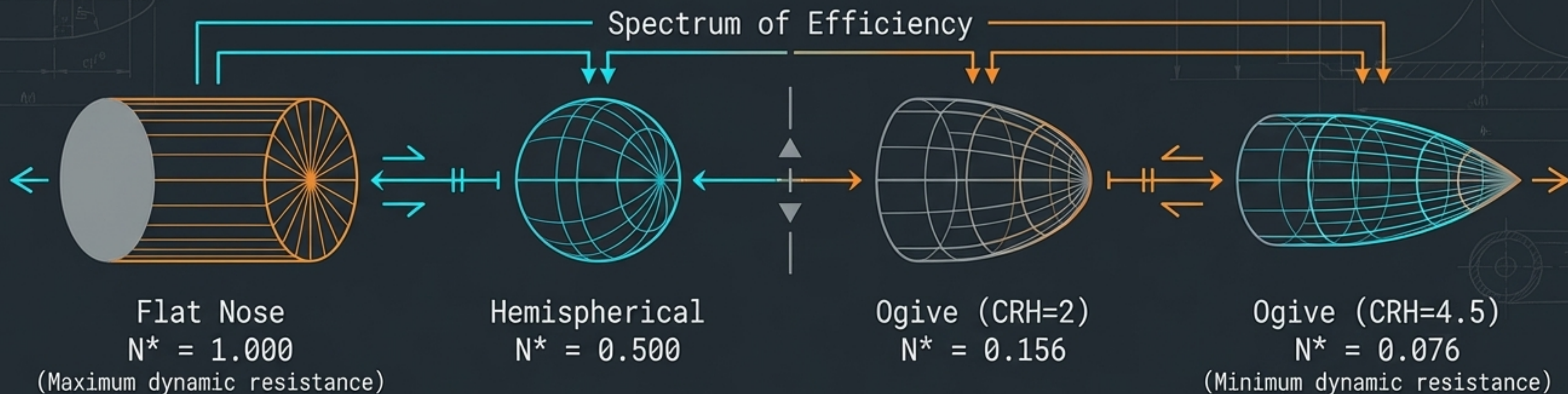
The inertial resistance of accelerating the concrete mass outward. Highly dependent on velocity and nose geometry (N^*).



Insight: At standard ballistic velocities (e.g., 277 m/s), the Static term dominates ~94% of the resistance force.

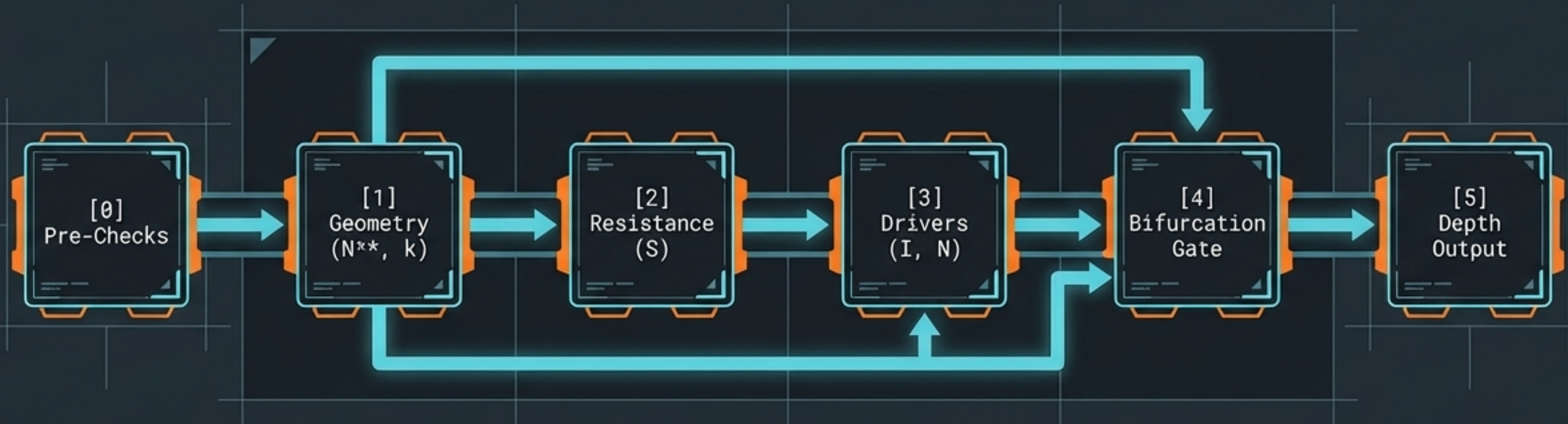
N*: The Spectrum of Geometric Efficiency

$$N^* = \frac{8}{d^2} \int_0^h \frac{y(y')^3}{1+(y')^2} dx$$



Key Insight: N^* is a continuous mathematical integral, not a discrete guess. A lower number precisely dictates a sharper, more efficient penetrator that minimizes inertial drag.

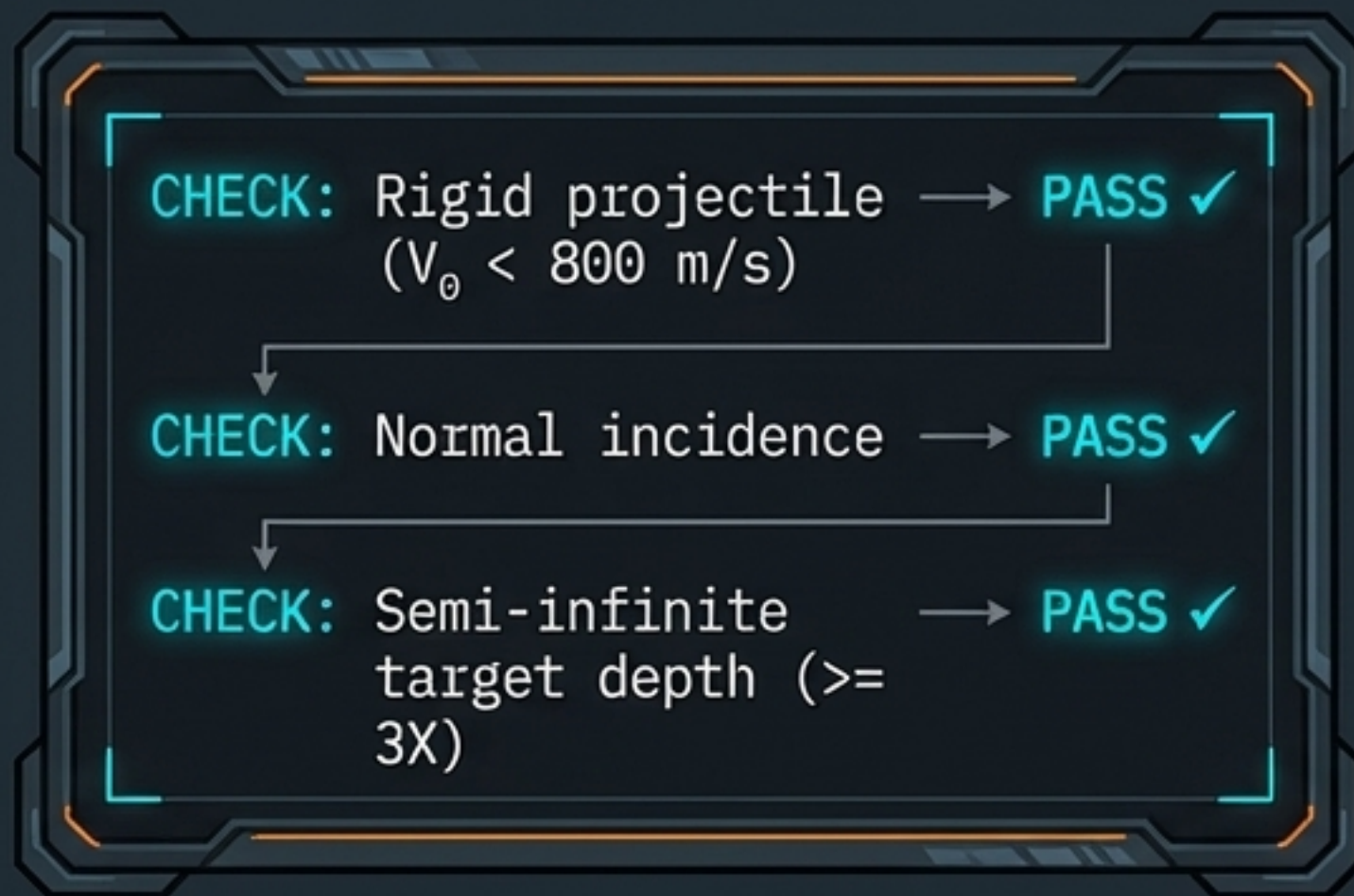
The A4 Pipeline: Executable Architecture



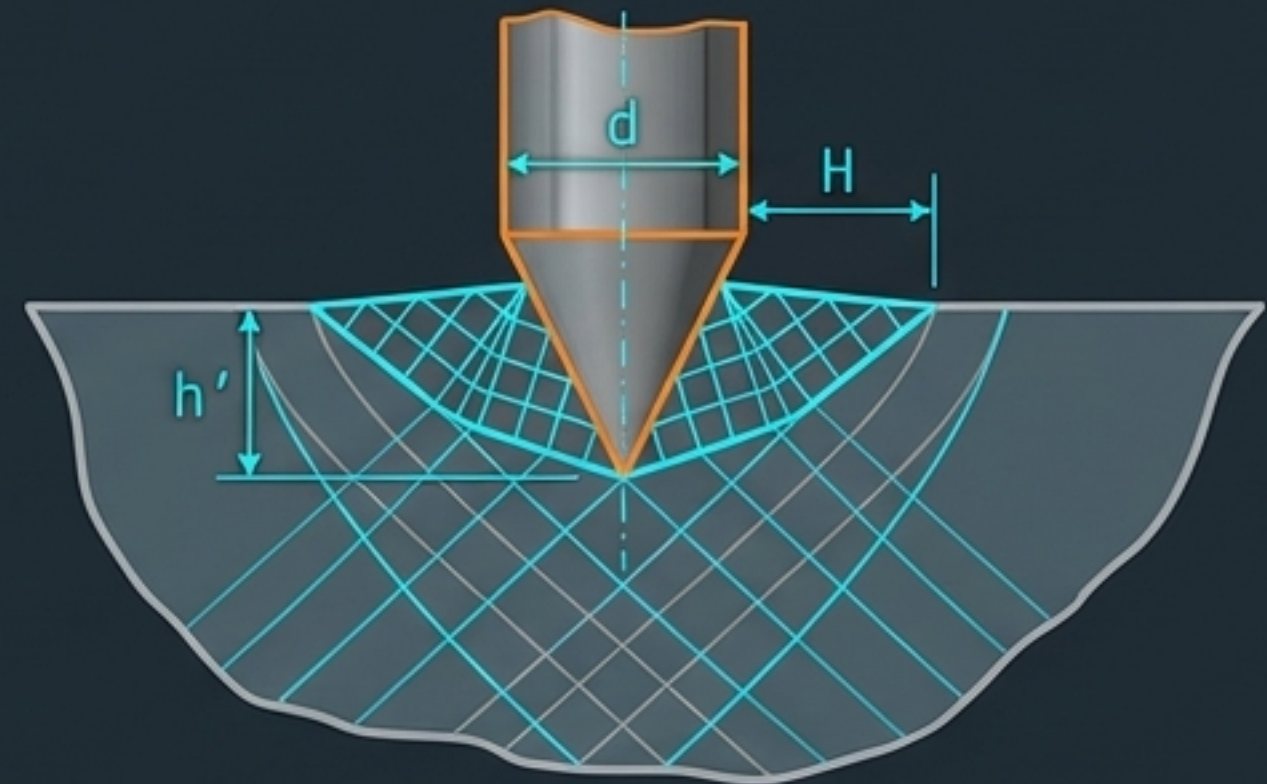
The A4 Pipeline translates the dual-theory physics into a deterministic, step-by-step computational engine.

Nodes 0 & 1: Boundaries and Geometry

[0] Pre-Checks (The Guardrails)



[1] Geometry & The Crater Limit (k)



Calculates N^* (Geometric Efficiency).

Calculates k (Dimensionless Crater Limit).

Based on Prandtl's plasticity theory, the depth is the flat-nose slip line $(0.707d)$ plus the nose length (H) .

$$k = 0.707 + H/d$$

Node 2: Dynamic Target Resistance (S)



Concept

Uniaxial lab tests (f_c) do not reflect how concrete behaves under dynamic, triaxial confinement during a ballistic strike.

The Calculation

$$S = 72.0 * f_c^{(-0.5)}$$

Insight

S acts as the crucial empirical bridge linking static material properties to dynamic cavity expansion resistance. An S value of 12 means the concrete effectively resists with 12x its standard lab strength during the impact.

Node 3: The Dimensionless Drivers (I and N)

Transformation from Buckingham Pi inputs into Pipeline A4 operational drivers.

The Impact Function (I)

$$I = \frac{I^*}{S}$$
$$= \frac{M * V_0^2}{S * d^3 * f_c}$$

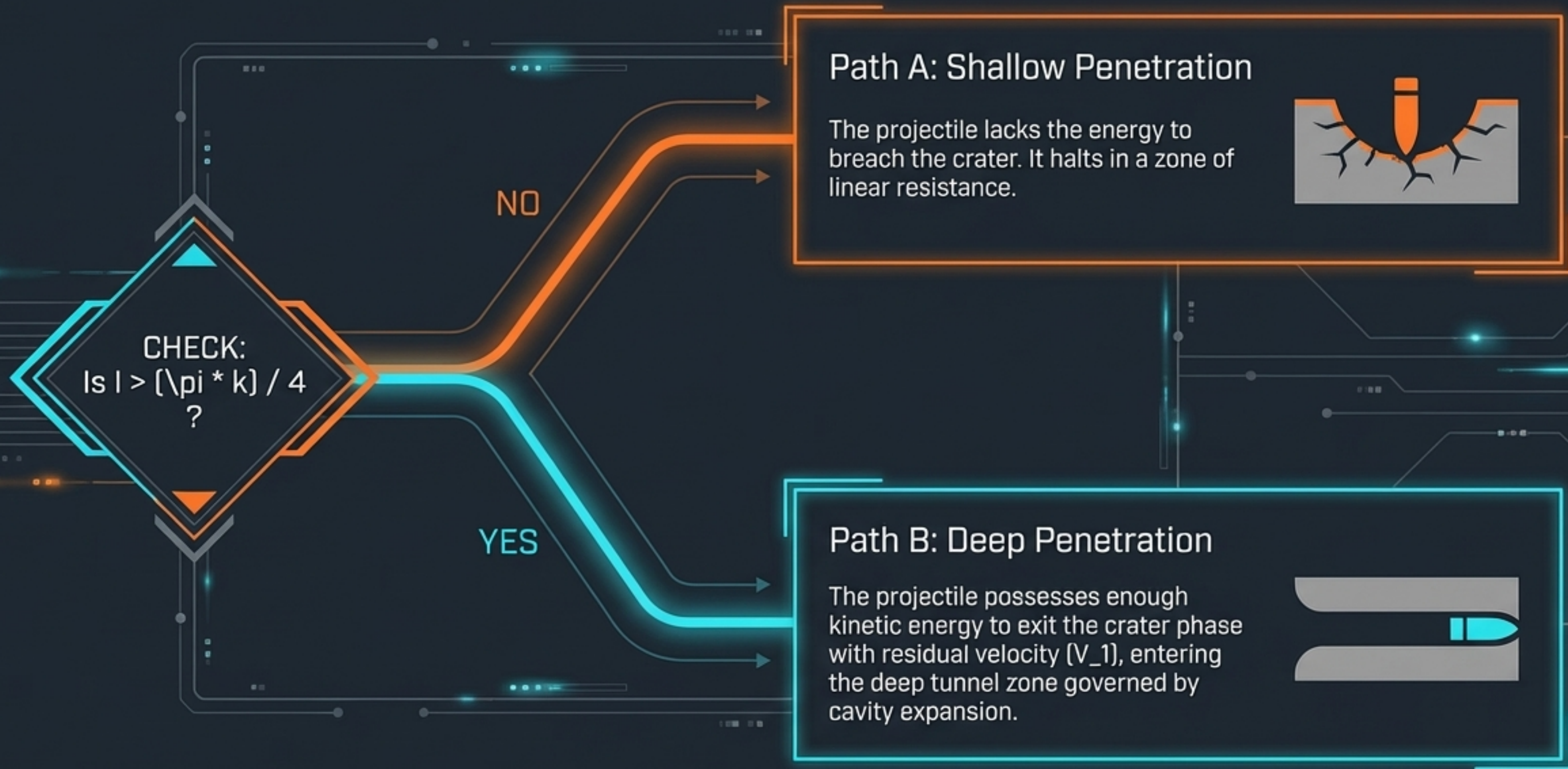
Meaning: The ratio of available kinetic energy to the target's actual confined absorption capacity. The ultimate measure of deep penetration potential.

The Geometry Function (N)

$$N = \frac{\lambda}{N^*}$$
$$= \frac{M}{N^* * \rho_c * d^3}$$

Meaning: Combines relative mass with nose sharpness. A heavy, highly sharp projectile yields a massive N value.

Node 4: The Bifurcation Gate



Node 5: Final Depth Computation

Shallow Regime (Crater, $X/d \leq k$)

$$X/d = \sqrt{\left\{ \left(\frac{4*k*I}{\pi} \right) * \left(\frac{1 + (k*\pi i)/(4*N)}{1 + I/N} \right) \right\}}$$

Deep Regime (Tunnel, $X/d > k$)

$$X/d = \left(\frac{2*N}{\pi} \right) * \ln\left(\frac{1 + (I/N - (\pi*k)/(4*N))}{1 + (\pi*k)/(4*N)} \right) + k$$

Key Physics Note: For deep penetration, depth is a logarithmic function of initial velocity, heavily scaled by the impact function I and offset by the crater depth k .

Verification: Introducing Shot 14

Spec Sheet

Projectile Specs

Material: **4340 Steel**

Geometry: **Ogive, CRH = 2.0**
 $N^* = 0.156$

Mass (M): **0.906 kg**

Velocity (V_0): **277 m/s**

Target Specs

Concrete Strength (f_c):
35.2 MPa

Density (ρ_c): **2370 kg/m³**

(Reference: Forrestal et al., 1994)

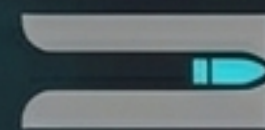
Pipeline A4 Inputs Generated

Resistance **$S = 12$**

Impact Driver **$l = 8.455$**

Geometry Driver **$N = 125.9$**

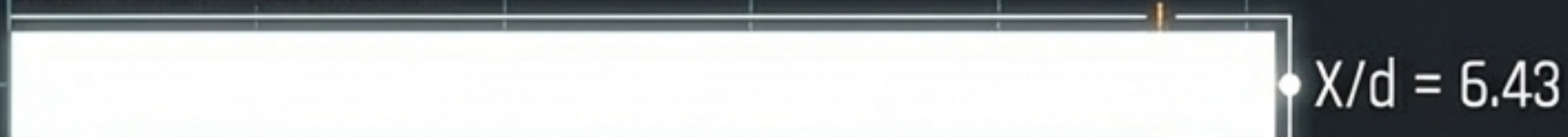
Bifurcation =
Deep Penetration
($l \gg \pi k/4$)



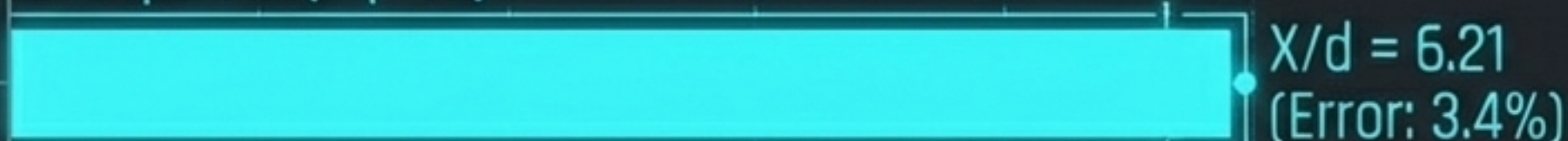
Verification: Shot 14 Results

Penetration Depth (X/d)

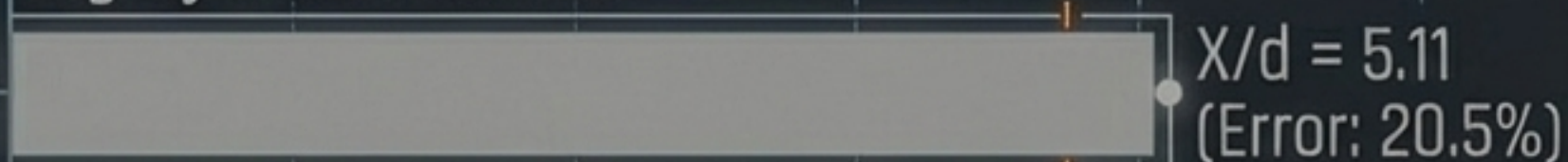
Actual Test Result



A4 Pipeline (Eq 15b)



Legacy NDRC Formula



Penetration Depth (X/d)

Analytical Insight

At 277 m/s, static resistance accounts for 94% of the forces. The NDRC formula fails to scale this deep-penetration regime regime accurately. The dimensionless A4 list model captures it with near-perfect precision.

Synthesis: The Dimensionless Advantage

A4 Pipeline

[0]
Input
Parameters

[1]
Resistance
Calc (S)

[2]
Impact
Driver Calc
(I)

[3]
Geometry
Driver Calc
(N)

[4]
Bifurcation
Check

[5]
Depth
Output

SHOT 14 RESULT: $X/d = 6.21$

Mathematically Robust

Eliminates the unit-dependency and discrete geometric flaws of the mid-century models.

Physically Grounded

Unifies Buckingham Pi dimensional analysis with the micro-physics of dynamic cavity expansion.

Computationally Actionable

Pipeline A4 provides a deterministic, highly accurate, six-step architecture ready for software integration.